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ON THE INDICATRIX BUNDLE ENDOWED WITH THE K-CONNECTION OVER A FINSLER SPACE.

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Introduction. Let us consider an n-dimensional Finsler space M with a fundamental function $F(x, y)$ and Cartan connection $\Gamma^{*}{}^{i}_{jk}(x, y)$. Then we can construct the indicatrix bundle $L = \underset{x \in M}{U} I_x$ over M, I_x being the indicatrix at a point x of M, and introduce in a natural way a metric on L, which, in fact, corresponds to the O-lift in [4][1]. On the other hand, though the choice of metrical connections on L is highly arbitrary, it seems to the present author that, for the present, it is enough for a practical use to consider two connections. One is the D-connection due to A. Deicke [1] and another is the K-connection due to M. Kurita [3].

In the papers [8], [9], [10], we treated with the indicatrix bundle endowed with the D-connection. In the present paper, we shall study the indicatrix bundle endowed with the K-connection. The terminologies and notations are referred to the papers [9], [10] unless otherwise stated.

§1. Metric and connection on L. Let (ω^A) $(A = 1, 2, \cdots 2n-1)$ be an adapted orthogonal coframe and (e_A) the adapted orthogonal frame dual to (ω^A). Then they are given by

$$(1.1)\quad \omega^a = \zeta^a_i dx^i \ (a = 1, 2, \cdots, n), \ \omega^{n+\alpha} = \omega^{(\alpha)} = \zeta^\alpha_i Dl^i \ (\alpha = 1, 2, \cdots, n-1),$$

$$(1.2)\quad e_a = \zeta^i_a \ (\partial/\partial x^i - N^j_i \partial/\partial l^j), \ e_{n+\alpha} = e_{(\alpha)} = \zeta^i_\alpha \partial/\partial l^i,$$

together with

$$(1.3)\quad g_{ij} = \sum_a \zeta^a_i \zeta^a_j, \ \zeta^n_i = l_i = \partial F/\partial y^i, \ g^{ij} = \sum_a \zeta^i_a \zeta^j_a,$$

1) Numbers in brackets refer to the references at the end of the paper.

$$\zeta_n^i = l^i = g^{ij} l_j, \quad Dl^i = dl^i + N^i_j dx^j, \quad N^i_j = \Gamma^*{}_{o\ j}^{\ i}.$$

From now on, we use indices as follows : Small Latin indices $a, b, c, \cdots; i, j, k, \cdot$ run from 1 to n and capital indices $A, B, C, \cdots$ from 1 to $2n-1$, while Greek indices $\alpha, \beta, \gamma, \cdots$ run from 1 to $n-1$.

A metric on L is given by the tensor G whose components are δ_{AB} with respect to (e_A). If we denote the inner product by $<\ ,\ >$, we have

$$(1.4) \quad < e_A, e_B > = \delta_{AB}, \quad < \omega^A, \omega^B > = \delta^{AB}, \quad ds^2 = \sum_A \omega^A \omega^A \ (s\ ; arc\ length).$$

Let L be endowed with the K-connection due to M. Kurita [3], [7]. This connection is defined by

$$\Gamma = (\omega^A_B) = \begin{bmatrix} \omega^a_b & \omega^a_{(\beta)} \\ \omega^{(\alpha)}_b & \omega^{(\alpha)}_{(\beta)} \end{bmatrix}, \quad \begin{array}{l} \omega^A_B = -\omega^B_A, \ \omega^{(\alpha)}_{(\beta)} = \omega^\alpha_\beta, \\ \omega^{(\alpha)}_b = \omega^a_{(\beta)} = O, \end{array}$$

$$(1.5) \quad \omega^A_B = \Gamma_{B\ C}^{\ A}\, \omega^C + \Gamma_{B(\gamma)}^{\ A}\, \omega^{(\gamma)}, \quad \Gamma_{b\ c}^{(\alpha)} = \Gamma_{b(\gamma)}^{(\alpha)} = \Gamma_{(\beta)\ c}^{\ a} = \Gamma_{(\beta)(\gamma)}^{\ a} = O,$$

$$\Gamma_{b\ c}^{\ a} = -\zeta_i^a{}_{,j}\, \zeta_b^i \zeta_c^j, \quad \Gamma_{b\ (\gamma)}^{\ a} = -\zeta_i^a |_j \zeta_b^i \zeta_\gamma^j,$$

where $_{,j}$ and $|_j$ indicate the first and second covariant differentiations of Cartan. The K-connection is metrical but not symmetric in general.

§2. Torsion and curvature.

The equations of structure are given by

$$(2.1) \quad \begin{array}{l} d\omega^a = \omega^b \wedge \omega^a_b + \omega^{(\beta)} \wedge \mu^{\ a}_{(\beta)}, \\ d\omega^{(\alpha)} = \omega^a \wedge \nu^{(\alpha)}_a + \omega^{(\beta)} \wedge \omega^{(\alpha)}_{(\beta)} + \frac{1}{2} Z^{(\alpha)}{}_{bc}\, \omega^b \wedge \omega^c, \end{array}$$

where

$$(2.2) \quad \begin{array}{l} \mu^{\ a}_{(\beta)} = A_j{}^i{}_k\, \zeta^j_\beta \zeta^a_i \zeta^k_b\, \omega^b, \quad \nu^{(\alpha)}_a = A_j{}^i{}_{k\,|\,o}\, \zeta^j_a \zeta^\alpha_i \zeta^k_\gamma\, \omega^{(\gamma)}, \\ Z^{(\alpha)}{}_{bc} = R_o{}^i{}_{jk}\, \zeta^\alpha_i \zeta^j_b \zeta^k_c, \quad A_{ijk} = \frac{1}{2} F\, \partial g_{ij} / \partial y^k, \quad A_j{}^i{}_k = g^{is} A_{jsk}. \end{array}$$

The torsion form τ^A and tensor $T_B{}^A{}_C$ are given by

$$(2.3) \quad \tau^A = d\omega^A - \omega^B \wedge \omega^A_B = \frac{1}{2} T_B{}^A{}_C\, \omega^B \wedge \omega^C \quad (T_B{}^A{}_C + T_C{}^A{}_B = O).$$

Then in virtue of (1.5), (2.1), (2.2), and (2.3) we have

$$\tau^a = \omega^{(\beta)} \wedge \mu^{\ a}_{(\beta)}, \quad \tau^{(\alpha)} = \omega^a \wedge \nu^{(\alpha)}_a + \frac{1}{2} Z^{(\alpha)}{}_{bc}\, \omega^b \wedge \omega^c,$$

$$(2.4)\quad T_b{}^a{}_c = T_{(\beta)}{}^a{}_{(\gamma)} = T_b{}^{(\alpha)}{}_c = T_{(\beta)}{}^{(\alpha)}{}_{(\gamma)} = O,\ T_{(\gamma)}{}^a{}_b = -A_b{}^a{}_{(\gamma)} = A_j{}^i{}_k\,\zeta_i^a\,\zeta_b^j\,\zeta_\gamma^k,$$

$$-T_c{}^{(\alpha)}{}_b = T_b{}^{(\alpha)}{}_c = R_{o}{}^i{}_{jk}\,\zeta_i^\alpha\,\zeta_b^j\,\zeta_c^k,\ T_b{}^{(\alpha)}{}_{(\gamma)} = A_j{}^i{}_{k\,|\,o}\,\zeta_i^\alpha\,\zeta_b^j\,\zeta_\gamma^k.$$

Then we can state

Proposition 1. *The K-connection is symmetric if and only if M is a locally Euclidean space. A path in L with respect to the K-connection does not coincide with an extremal in L.*

Proof. If $T_B{}^A{}_C = O$, it follows from (2.4) that $A_j{}^i{}_k = O$ and $R_j{}^i{}_{kh} = O$ and vice versa. A path coincides with an extremal if and only if $T_B{}^A{}_C$ are skew-symmetric in all indices A, B and C, while (2.4) denies the latter.

Let Ω_B^A and $K_B{}^A{}_{CD}$ be the curvature form and tensor. Then they are defined by

$$(2.5)\quad \Omega_B^A = \omega_B^C \wedge \omega_C^A - d\omega_B^A = \frac{1}{2} K_B{}^A{}_{CD}\,\omega^C \wedge \omega^D \quad (K_B{}^A{}_{CD} = -K_B{}^A{}_{DC}),$$

which is reducible to

$$(2.6)\quad \Omega_B^A = \frac{1}{2} R_B{}^A{}_{cd}\,\omega^c \wedge \omega^d + P_B{}^A{}_{c(\sigma)}\,\omega^c \wedge \omega^{(\sigma)} + \frac{1}{2} S_B{}^A{}_{(\gamma)(\sigma)}\,\omega^{(\gamma)} \wedge \omega^{(\sigma)}.$$

Calculating the second term of (2.5) on use of (1.1), (1.3), (1.5), (2.1), (2.2) and the Ricci identities [6], and comparing with the right hand side of (2.6), we have (cf. [7])

$$(\mathrm{C}\ 1)\quad R_b{}^a{}_{cd} = R_j{}^i{}_{kh}\,\zeta_i^a\,\zeta_b^j\,\zeta_c^k\,\zeta_d^h,\quad P_b{}^a{}_{c(\sigma)} = P_j{}^i{}_{kh}\,\zeta_i^a\,\zeta_b^j\,\zeta_c^k\,\zeta_\sigma^h,$$
$$S_b{}^a{}_{(\gamma)(\sigma)} = S_j{}^i{}_{kh}\,\zeta_i^a\,\zeta_b^j\,\zeta_\gamma^k\,\zeta_\sigma^h,$$

$$(\mathrm{C}\ 2)\quad R_{(\beta)}{}^a{}_{cd} = P_{(\beta)}{}^a{}_{c(\sigma)} = S_{(\beta)}{}^a{}_{(\gamma)(\sigma)} = O,$$

$$(\mathrm{C}\ 3)\quad R_b{}^{(\alpha)}{}_{cd} = P_b{}^{(\alpha)}{}_{c(\sigma)} = S_b{}^{(\alpha)}{}_{(\gamma)(\sigma)} = O,$$

$$(\mathrm{C}\ 4)\quad R_{(\beta)}{}^{(\alpha)}{}_{cd} = R_j{}^i{}_{kh}\,\zeta_i^\alpha\,\zeta_\beta^j\,\zeta_c^k\,\zeta_d^h,\quad P_{(\beta)}{}^{(\alpha)}{}_{c(\sigma)} = P_j{}^i{}_{kh}\,\zeta_i^\alpha\,\zeta_\beta^j\,\zeta_c^k\,\zeta_\sigma^h,$$
$$S_{(\beta)}{}^{(\alpha)}{}_{(\gamma)(\sigma)} = (S_j{}^i{}_{kh} + h_{jk}\,h_h^i - h_{jh}\,h_k^i)\ \zeta_i^\alpha\,\zeta_\beta^j\,\zeta_\gamma^k\,\zeta_\sigma^h,$$

where $R_j{}^i{}_{kh}$, $P_j{}^i{}_{kh}$ and $S_j{}^i{}_{kh}$ are the first, second and third curvature tensors of Cartan.

If we denote by $\overset{m}{R}_j{}^i{}_{kh}$, $\overset{m}{P}_j{}^i{}_{kh}$ and $\overset{m}{S}_j{}^i{}_{kh}$ the tensors on M determined by (C m) (m = 1, 2, 3, 4) respectively, we obtain the following :

$$(2.7)\quad \overset{1}{R}{}_j{}^i{}_{kh}=R_j{}^i{}_{kh},\quad \overset{1}{P}{}_j{}^i{}_{kh}=P_j{}^i{}_{kh},\quad \overset{1}{S}{}_j{}^i{}_{kh}=S_j{}^i{}_{kh},$$

$$(2.8)\quad \overset{2}{R}{}_j{}^i{}_{kh}=l_j R_o{}^i{}_{kh},\quad \overset{2}{P}{}_j{}^i{}_{kh}=l_j P_o{}^i{}_{kh},\quad \overset{2}{S}{}_j{}^i{}_{kh}=l_j T^i{}_{kh},$$

$$(2.9)\quad \overset{3}{R}{}_j{}^i{}_{kh}=l^i R_j{}^o{}_{kh},\quad \overset{3}{P}{}_j{}^i{}_{kh}=l^i P_j{}^o{}_{kh},\quad \overset{3}{S}{}_j{}^i{}_{kh}=l^i T_{jkh},$$

$$(2.10)\quad \begin{aligned}\overset{4}{R}{}_j{}^i{}_{kh}&=R_j{}^i{}_{kh}-R_j{}^o{}_{kh}l^i-R_o{}^i{}_{kh}l_j,\\ \overset{4}{P}{}_j{}^i{}_{kh}&=P_j{}^i{}_{kh}-P_j{}^o{}_{kh}l^i-P_o{}^i{}_{kh}l_j,\\ \overset{4}{S}{}_j{}^i{}_{kh}&=S_j{}^i{}_{kh}+h_{jk}h^i_h-h_{jk}h^i_k,\end{aligned}$$

where $T^i{}_{kh}$ is any homogeneous indicatory tensor of degree 0 in l^i, provided $T^i{}_{kh}=-T^i{}_{hk}$.

Immediately we have

Proposition 2. *The curvature tensor $K_B{}^A{}_{CD}$ on L never vanishes.*

Proof. If $K_B{}^A{}_{CD}=0$, it follows from (2.7) and (2.10) that

$$h_{jk}h^i_k-h_{jh}h^i_k=0,$$

which implies $h_{jk}=0$ and hence the rank of (g_{ij}) is less than n, contrary to hypothesis.

§3. Covariant differentiations and distributions on L. For the sake of brevity, we consider only a proper tensor of type (1 . 1) on L whose components are $T^A_B\,(x,\,l)$ with respect to (e_A). The covariant differentials of T^A_B are given by

$$DT^A_B=dT^A_B+\omega^A_D T^D_B-\omega^D_B T^A_D=\nabla_D T^A_B\,\omega^D,$$

where $\nabla_D T^A_B$ are covariant derivatives and the camponents of a tensor of type (1 , 2) on L. For $\nabla_D T^A_B$, we have

$$(3.1)\quad \begin{aligned}&\nabla_a T^A_B=\partial_a T^A_B-T^A_{B\,\|\,i}N^i_j\zeta^j_a+\Gamma_D{}^A{}_a T^D_B-\Gamma_B{}^D{}_a T^A_D,\\ &\nabla_{(\alpha)}T^A_B=\partial_{(\alpha)}T^A_B+\Gamma_D{}^A{}_{(\alpha)}T^D_B-\Gamma_B{}^D{}_{(\alpha)}T^A_D,\\ &\partial_a T^A_B=\zeta^i_a\,\partial T^A_B/\partial x^i,\quad \nabla_{(\alpha)}=\nabla_{n+\alpha},\quad \partial_{(\alpha)}T^A_B=T^A_{B\,\|\,i}\zeta^i_\alpha.\end{aligned}$$

Let T be a tensor on L whose components are given by

$$(3.2)\quad T^A_B=\begin{bmatrix}T^a_b & T^a_{(\beta)}\\ T^{\ b}_{(\alpha)} & T^{(\alpha)}_{(\beta)}\end{bmatrix},\quad \begin{aligned}T^a_b&=\overset{1}{T}{}^i_j\zeta^a_i\zeta^j_b,\quad T^a_{(\beta)}=\overset{2}{T}{}^i_j\zeta^a_i\zeta^j_\beta,\\ T^{(\alpha)}_b&=\overset{3}{T}{}^i_j\zeta^\alpha_i\zeta^j_b,\quad T^{(\alpha)}_{(\beta)}=\overset{4}{T}{}^i_j\zeta^\alpha_i\zeta^j_\beta,\end{aligned}$$

where $\overset{1}{T}{}^i_j$, $\overset{2}{T}{}^i_j$, $\overset{3}{T}{}^i_j$ and $\overset{4}{T}{}^i_j$ are tensors on M. Then, from (1.5) and (3.1) we have

$$\nabla_c T^a_b = \overset{1}{T}{}^i_{j\,|\,k}\, \zeta^a_i \zeta^j_b \zeta^k_c, \quad \nabla_c T^a_{(\beta)} = \overset{2}{T}{}^i_{j\,|\,k}\, \zeta^a_i \zeta^j_\beta \zeta^k_c,$$

$$\nabla_c T^{(\alpha)}_b = \overset{3}{T}{}^i_{j\,|\,k}\, \zeta^\alpha_i \zeta^j_b \zeta^k_c, \quad \nabla_c T^{(\alpha)}_{(\beta)} = \overset{4}{T}{}^i_{j\,|\,k}\, \zeta^\alpha_i \zeta^j_\beta \zeta^k_c$$

$$(3.3) \quad \nabla_{(\gamma)} T^a_b = \overset{1}{T}{}^i_j\,|_k\, \zeta^a_i \zeta^j_b \zeta^k_\gamma, \quad \nabla_{(\gamma)} T^a_{(\beta)} = \overset{2}{T}{}^i_j\,|_k\, \zeta^a_i \zeta^j_\beta \zeta^k_\gamma - \delta_{\beta\gamma} \overset{2}{T}{}^i_o \zeta^a_i,$$

$$\nabla_{(\gamma)} T^{(\alpha)}_b = \overset{3}{T}{}^i_j\,|_k\, \zeta^\alpha_i \zeta^j_b \zeta^k_\gamma - \delta^\alpha_\gamma \overset{3}{T}{}^o_j \zeta^j_b,$$

$$\nabla_{(\gamma)} T^{(\alpha)}_{(\beta)} = \overset{4}{T}{}^i_j\,|_k\, \zeta^\alpha_i \zeta^j_\beta \zeta^k_\gamma - \delta^\alpha_\gamma \overset{4}{T}{}^o_j \zeta^j_\beta - \delta_{\beta\gamma} \overset{4}{T}{}^i_o \zeta^\alpha_i.$$

Let $X = u^A e_A$ and $Y = v^A e_A$ be any two vector fields on L. Then if we denote by ∇_X the covariant differentiation in the direction of X, it follows from (1.2) and (1.5) that

$$(3.4) \quad \nabla_X Y = (\delta_B v^D + v^A \Gamma_A{}^D{}_B)\, u^B e_D, \qquad \delta_a v^D = \partial_a v^D - v^D{}_{\|i} N^i_j \zeta^j_a, \quad \delta_{(\alpha)} v^D = \partial_{(\alpha)} v^D.$$

A distribution E on L is said to be *parallel* if, for any vector field X on L and any vector field Y belonging to E, $\nabla_X Y$ belongs always to E.

Let V be an n-dimensional distribution on L defined by $\omega^{(\alpha)} = 0$. Then it is known that M is realizable as V such that the metric and connection on V induced from those on L identify with the metric and connection on M, and that a local base for V is given by (e_a), being also a local base for M. From (1.5) and (3.4) we have

$$(3.5) \quad \nabla_{e_A} e_a = \Gamma_a{}^b{}_A e_b + \Gamma_a{}^{(\beta)}{}_A e_{(\beta)} = \Gamma_a{}^b{}_A e_b, \qquad \nabla_X Y = (\delta_B v^d + v^a \Gamma_a{}^d{}_B)\, u^B e_d,$$

where $Y = v^a e_a$ and $v^{(\alpha)} = 0$.

Let I be an $(n\text{-}1)$-dimensional distribution on L defined by $\omega^a = 0$. Then I is involutive and the orthogonal complement of V, too. And the indicatrix I_x at any fixed point x of M is regarded as an integral manifold of I, the local equation of I_x being $x^i = const.$, and a local base for I_x is given by $(e_{(\alpha)})$. In the same way as before we have

$$\nabla_{e_A} e_{(\alpha)} = \Gamma_{(\alpha)\,A}^{\;\;b} e_b + \Gamma_{(\alpha)\,A}^{(\beta)} e_{(\beta)} = \Gamma_{(\alpha)\,A}^{(\beta)} e_{(\beta)},$$
(3 . 6)
$$\nabla_X Y = (\delta_A v^{(\alpha)} + v^{(\beta)} \Gamma_{(\beta)\,A}^{(\alpha)})\, u^A e_{(\alpha)},$$

where $Y = v^{(\alpha)} e_{(\alpha)}$ and $v^a = O$. In virtue of (3 . 5) and (3 . 6) we can state

Theorem 1. *The distributions V and I are both parallel in L.*

Any indicatrix I_x becomes a Riemannian submanifold of L by means of the metric and connection on I_x induced from those on L and the differential geometry to be developed on I_x is the same as that in §2 of [9].

I_x is called an *auto-parallel* submanifold of L if, for any vector fields X and Y on I_x, $\nabla_X Y$ is tangential to I_x at every point of I_x [2].

I_x is called to be *h-parallel* in L along a curve C in M if the h-mapping along C is the parallel displacement with respect to the K-connection [9]. From Theorem 1 we have

Corollary 1. 1. *Any indicatrix I_x is an auto-parallel submanifold of L. Any indicatrix I_x is h-parallel in L along any curve in M.*

V is called to be *auto-parallel* in L if, for any vector fields X and Y belonging to V, $\nabla_X Y$ belongs to V.

V is called to be *v-parallel* along an indicatrix I_x if, for any vector field Y belonging to V and any vector field X on I_x, $\nabla_X Y$ belongs to V. From Theorem 1 we have

Corollary 1. 2. *The distribution V is auto-parallel in L and v-parallel along any indicatri I_x.*

§4. **Curves in L.** Let C be a curve in L defined by

(4 . 1) $C : x^i = x^i(s), l^i = l^i(s)$ (*s ; are length*),

provided $F(x, l) = 1$.

Putting $X = (\omega^A/ds)\, e_A$, from (1 . 5) and (3 . 6) we have

$$\nabla_X X = \{d(\omega^a/ds)/ds + (\omega^b/ds)(\omega^B/ds)\Gamma_{b\ B}^{\ a}\}\, e_a \tag{4 . 2}$$
$$+ \{d[\omega^{(\alpha)}/ds]/ds + [\omega^{(\beta)}/ds](\omega^B/ds)\Gamma_{(\beta)\,B}^{(\alpha)}\}\, e_{(\alpha)},$$

which is, in virtue of (1 . 1), (1 . 3) and (1. 5), reducible to

$$\nabla_X X = \xi^i \zeta_i^a e_a + \eta^i \zeta_i^\alpha e_{(\alpha)},$$

$$\xi^i = d^2x^i/ds^2 + \Gamma^{*\ i}_{j\ k}(dx^j/ds)(dx^k/ds) + A_j{}^i{}_k(dx^j/ds)(Dl^k/ds) \tag{4 . 3}$$

$$\eta^i = h^i_s \ \{d\ (Dl^s/ds)/ds + \Gamma^{*}{}_j{}^s{}_k\ (dx^j/ds)\ (Dl^k/ds)\}$$
$$+ A_j{}^i{}_k\ (Dl^j/ds)\ (Dl^k/ds)$$

Hence, from (4 . 3) we have

Theorem 2. *An equation of a path C in L is given by*

(4 . 4) $\xi^i = 0, \quad \eta^i = 0$ *in (4 . 3).*

Let $\overline{C}$: $x^i = x^i\ (s)$ (s ; *arc· lenth*) be a curve in *M*. Then if *C* is a horizontal lift of $\overline{C}$ to *L*, the following holds good along $\overline{C}$:

(4 . 5) $\omega^{(\alpha)}/ds = 0$ or $Dl^i/ds = 0$.

Applying (4 . 5) to (4 . 3), we get

(4 . 6) $\nabla_X X = \xi^i\ \zeta^a_i\ e_a, \quad \xi^i = d^2x^i/ds^2 + \Gamma^{*}{}_j{}^i{}_k\ (dx^j/ds)\ (dx^k/ds).$

Hence, because of (4 . 6) we can state

Corollary 2. 1. *If a horizontal lift C of a curve* $\overline{C}$ *in M to L is a path in L, then the curve* $\overline{C}$ *is a geodesic in M. Conversely, any holizontal lift of a geodesic in M is a path in L.*

Let C^* : $l^i = l^i\ (s)$ (s ; *arc length*) be a curve in an indicatrix I_x. Then if *C* is tangential to I_x along C^*, the following holds good :

(4 . 7) $\omega^a/ds = 0$ or $dx^i/ds = 0.$

Applying (4 . 7) to (4 . 3), we obtain

(4 . 8) $\nabla_X X = \eta^i\ \zeta^\alpha_i\ e_{(\alpha)}, \quad \eta^i = h^i_s\ (d^2 l^s/ds^2) + A_j{}^i{}_k\ (dl^j/ds)\ (dl^k/ds).$

In this case, since $h^i_s\ (d^2 l^s/ds^2) = d^2 l^i/ds^2 + l^i$, it follows from (4 . 8) that $\nabla_X X = 0$ if and only if

(4 . 9) $d^2 l^i/ds^2 + l^i + A_j{}^i{}_k\ (dl^j/ds)\ (dl^k/ds) = 0,$

which is the equation of a geodesic C^* in I_x [5], [10].

Thus we have

Corollary 2. 2. *A path C in L satisfying (4 . 7) is a geodesic in some indicatrix I_x. Every geodesic in any indicatrix I_x is a path in L.*

References

[1] Deicke, A. Finsler spaces as non-holonomic subspace. J. London Math. Soc. 30 1955, 53-58.

[2] Kobayashi, S. and Nomizu, K. Foundation of differential geometry Vol. II. Interscience Publ., 1969.

[3] Kurita, M. Theory of Finsler spaces based on the contact structure. J. Math. Soc. Japan, 18 1966, 119-134.

[4] Matsumoto, M. Foundation of Finsler geometry and special Finsler spaces. VEB Deutscher Verlag der Wissenschaften, Berlin, 1977.

[5] Matsumoto, M. C-geodesics in tangent spaces, to appear.

[6] Rund, H. The differential geometry of Finsler spaces. Grundlehr. Math. Wiss. 101, Springer Verlag, 1959.

[7] Yasuda, H. Finsler spaces as distributions on Riemannian manifolds. Hokkaido Math. J. 1 1972, 280-297.

[8] Yasuda, H. On Finsler spaces with absolute parallelism of line-elements. J. Korean Math. Soc. 13 1976, 179-188.

[9] Yasuda, H. On the indicatrices of a Finsler space. to appear in Tensor N. S. 1979.

[10] Yasuda, H. and Yamanoi, T. On curves in the indicatrix bundle over a Finsler space. to appear in Tensor N. S. 1979.

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